

Analysis of the Energy Spectra of Ground States of Deformed Nuclei in rare-earth region *

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Abstract: The ${}_{62}Sm$, ${}_{64}Gd$, ${}_{64}Dy$, ${}_{70}Yb$, ${}_{72}Hf$ and ${}_{74}W$ nuclei are classified as deformed nuclei. Low-lying bands are one of the most fundamental excitation modes in the energy spectra of deformed nuclei. In this paper a theoretical analysis of the experimental data within the phenomenological model is presented. The energy spectra of ground states are calculated. It is found the low-lying spectra of ground band states are in good agreement with the experimental data.

Key words: ground state, energy spectra, rare - earth, deformed nuclei

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1 Introduction

Though the structure of rotational deformed nuclei and the nature of low excited levels in these nuclei studying already is 50 years, but these problem is rather actual and for today.

Nowadays, accumulated of rich experimental data on the excited and low excited states of deformed nuclei, which is need to development of their theoretical researches.

Phenomenological nuclear adiabatic model described by Bohr and Mottelson [1] has been played a big role in understanding the properties of deformed nuclei. According to this model, low excitation states of even-even deformed nuclei are connected with rotation of axial-symmetric nucleus as a whole. Such a simple phenomenological explanation allows a description of a large number of experimental data on deformed nuclei and predicts many new properties of these nuclei.

In the present paper the deviations from the adiabatic theory which appear in energies of ground band states is analyzed, within the phenomenological model [2, 3], which into account Coriolis mixture of low-lying state bands. The objects of calculation are ${}_{152-156}Sm$, ${}_{156-166}Gd$, ${}_{156-166}Dy$, ${}_{166-176}Yb$, ${}_{170-180}Hf$ and ${}_{174-184}W$, isotopes. These nuclei have been quite well studied experimentally such as in nuclear reactions and Coulomb excitation [4–6]. Experiments make the systematical measurement of properties of low-lying

states.

2 Energy spectra of *gr*- state bands

The energy of rotational core $E_{rot}(I)$ is in agreement with the energy of the ground state of rotational bands of even-even deformed nuclei in the lower value of spin I .

The effective angular frequency of rotating nucleus is defined as follows:

$$\omega_{eff} = \frac{E^{exp.}(I+1) - E^{exp.}(I-1)}{2} \quad (1)$$

and hence the effective moment of inertia for states $\mathfrak{I}_{eff}(I)$ in terms of the angular frequency of rotation $\omega_{rot}(I)$ is

$$\mathfrak{I}_{eff} = \frac{(I(I+1))^{1/2}}{\omega_{eff}(I)}. \quad (2)$$

From “Eq. (2)”, we can calculate the effective moment of inertia $\mathfrak{I}_{eff}(I)$. Nuclear angular frequency of rotation $\omega_{eff}(I)$ is obtained by “Eq. (1)”, with the energy $E^{exp.}(I)$ from experiment [7]. At low angular frequency of rotation, i.e. in low spin $I \leq 8\hbar$ the dependency is linear. We parameterize this dependency as follows:

$$\mathfrak{I}_{eff} = \mathfrak{I}_0 + \mathfrak{I}_1 \omega_{eff}^2(I). \quad (3)$$

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Equation “Eq. (3)” defines the parameters of inertia \mathfrak{I}_0 and \mathfrak{I}_1 , for the effective moment of inertia $\mathfrak{I}_{eff}(I)$ when $I \leq 8\hbar$.

The energy of the rotational ground band state is calculated using the parameters \mathfrak{I}_0 and \mathfrak{I}_1 by Harris parametrization for the energy and angular momentum [8].

$$E_{rot}(I) = \frac{1}{2}\mathfrak{I}_0\omega_{rot}^2(I) + \frac{3}{4}\mathfrak{I}_1\omega_{rot}^4(I). \quad (4)$$

3 Result and Discussion

To analyze the properties of states of the ground band in deformed nuclei, the phenomenological model of [2] has been utilized.

The Numerical values for the parameters \mathfrak{I}_0 and \mathfrak{I}_1 are determined using the least square method in “Eq. (3)”. These results are shown in Table 1, for the isotopes $^{152,154}Sm$, $^{158-162}Dy$ and $^{172-176}Yb$ respectively (\mathfrak{I}_0 in MeV^{-1} and in MeV^{-3}).

Table 1. The values of parameters \mathfrak{I}_0 , \mathfrak{I}_1 and E_{2+} (MeV) for isotopes Sm , Gdy , Dy , Yb , Hf and W , respectively.

A	\mathfrak{I}_0	\mathfrak{I}_1	$E_{2+}^{exp.}$	E_{2+}^{theory}	Q_0 Ref. [7]
<i>Sm</i>					
152	24.74	256.57	0.1218	0.1234	5.83 (057)
154	36.07	178.88	0.0724	0.0841	6.53 (15)
156	39.22	98.36	0.0760	0.0778	7.85 (79)
<i>Gd</i>					
156	38.74	95.25	0.0890	0.0769	6.76 (34)
158	37.52	107.00	0.0795	0.0795	7.03 (4)
160	39.72	83.49	0.0755	0.0752	7.16 (2)
<i>Dy</i>					
156	21.93	238.14	0.1370	0.1290	6.12 (6)
158	29.69	174.26	0.0989	0.0990	6.85 (8)
160	33.96	131.07	0.0898	0.0870	6.91 (20)
162	36.61	105.77	0.0806	0.3350	7.13
164	40.25	121.09	0.0734	0.0740	7.49
166	38.68	73.82	0.0766	0.0770	–
<i>Yb</i>					
166	28.75	132.20	0.1024	0.1032	7.26 (14)
168	33.34	148.77	0.0877	0.0890	7.62 (14)
170	35.06	87.82	0.0843	0.0850	7.80 (30)
172	37.60	81.31	0.0788	0.0790	7.91 (18)
174	38.71	76.65	0.0765	0.0770	7.82 (24)
176	36.00	63.96	0.0821	0.0830	7.59 (3)
<i>Hf</i>					
170	27.93	248.95	0.1008	0.1007	7.14 (30)
172	30.02	172.45	0.0953	0.0953	6.87 (18)
174	31.09	134.03	0.0900	0.0901	7.29 (24)
176	33.70	92.27	0.0884	0.0877	7.23 (8)
178	31.95	71.88	0.0932	0.0928	6.98 (4)
180	32.06	36.53	0.0933	0.0931	6.93 (3)
<i>W</i>					
174	25.06	212.67	0.1130	0.1136	–
176	25.91	177.61	0.1091	0.1097	–
178	26.87	143.47	0.1061	0.1069	–
180	27.51	127.14	0.1036	0.1030	6.24 (11)
182	28.31	110.50	0.1001	0.1045	6.57 (8)
184	29.22	98.36	0.1112	0.1015	6.27 (8)

The values of angular frequency and energy spectra of Sm , Dy and Yb is illustrated in Table 2, respectively. All of the predicted data seems with agreed with the experimental data.

Table 2. The Value of angular frequency and energy spectra of the Sm (E in MeV).

I	^{152}Sm			^{154}Sm			^{156}Sm		
	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$
2 ⁺	0.091	0.122	0.116	0.066	0.082	0.082	0.062	0.076	0.076
4 ⁺	0.147	0.366	0.360	0.116	0.267	0.268	0.111	0.250	0.251
6 ⁺	0.190	0.707	0.701	0.160	0.544	0.546	0.156	0.517	0.519
8 ⁺	0.225	1.126	1.119	0.197	0.903	0.904	0.197	0.872	0.874
10 ⁺	0.254	1.609	1.599	0.230	1.333	1.333	0.235	1.307	1.307
12 ⁺	0.280	2.149	2.134	0.260	1.826	1.824	0.269	1.819	1.812
I	^{156}Dy			^{158}Dy			^{160}Dy		
	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$
2 ⁺	0.101	0.138	0.129	0.080	0.099	0.100	0.071	0.087	0.087
4 ⁺	0.160	0.404	0.396	0.136	0.317	0.319	0.124	0.284	0.286
6 ⁺	0.204	0.770	0.763	0.183	0.638	0.640	0.171	0.581	0.584
8 ⁺	0.239	1.216	1.207	0.222	1.044	1.046	0.213	0.967	0.970
10 ⁺	0.268	1.725	1.716	0.255	1.520	1.525	0.249	1.429	1.433
12 ⁺	0.294	2.286	2.280	0.285	2.049	2.067	0.282	1.951	1.965
I	^{162}Dy			^{164}Dy			^{166}Dy		
	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$
2 ⁺	0.066	0.081	0.081	0.060	0.073	0.074	0.063	0.077	0.077
4 ⁺	0.117	0.266	0.268	0.107	0.242	0.244	0.113	0.254	0.255
6 ⁺	0.164	0.549	0.551	0.151	0.501	0.504	0.160	0.527	0.530
8 ⁺	0.206	0.921	0.924	0.190	0.844	0.846	0.203	0.892	0.894
10 ⁺	0.244	1.375	1.376	0.226	1.261	1.264	0.244	1.341	1.342
12 ⁺	0.279	1.901	1.900	0.258	1.745	1.749	0.281	1.868	1.867
I	^{166}Yb			^{168}Yb			^{170}Yb		
	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$
2 ⁺	0.083	0.102	0.103	0.072	0.088	0.089	0.069	0.084	0.085
4 ⁺	0.142	0.331	0.332	0.125	0.287	0.289	0.123	0.277	0.280
6 ⁺	0.193	0.668	0.670	0.172	0.585	0.589	0.172	0.574	0.577
8 ⁺	0.235	1.098	1.100	0.212	0.970	0.975	0.217	0.964	0.967
10 ⁺	0.272	1.606	1.610	0.247	1.426	1.435	0.257	1.438	1.442
12 ⁺	0.305	2.176	2.188	0.278	1.936	1.962	0.293	1.984	1.993
I	^{172}Yb			^{174}Yb			^{176}Yb		
	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$	$\omega_{rot}^{theor}(I)$	$E_{rot}^{exp.}(I)$	$E_{rot}^{theor}(I)$
2 ⁺	0.065	0.079	0.079	0.063	0.076	0.077	0.067	0.082	0.083
4 ⁺	0.116	0.260	0.262	0.113	0.253	0.255	0.121	0.272	0.274
6 ⁺	0.163	0.540	0.543	0.159	0.526	0.529	0.171	0.565	0.568
8 ⁺	0.207	0.912	0.914	0.203	0.890	0.892	0.217	0.955	0.958
10 ⁺	0.247	1.370	1.368	0.243	1.336	1.339	0.260	1.431	1.437
12 ⁺	0.283	1.907	1.899	0.279	1.861	1.862	0.299	1.985	1.998

The comparison between the calculated values of energy of the ground band state with experimental data [7] is given for the isotopes ^{62}Sm , ^{64}Gd , ^{66}Dy , ^{70}Yb , ^{72}Hf and ^{74}W in Figs. ?? Figs. 1–5, respectively. From the figures, we see that energy difference $\Delta E(I) = E^{theor}(I) - E^{exp.}(I)$, increases with the increase in the angular momentum I . This is due to the occurrence of the non-adiabaticity of energy rotational bands in large spin.

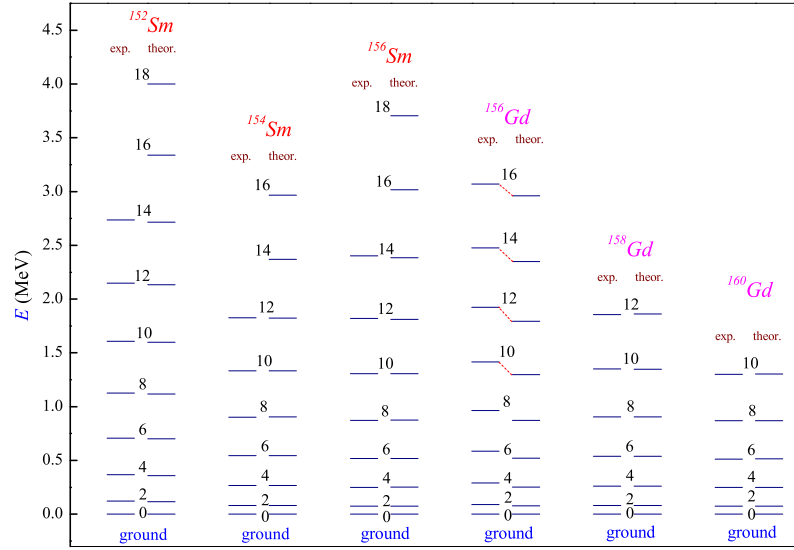


Fig. 1. The comparison experimental and theoretical spectra energy of ground bands for isotopes *Sm* and *Gd*, correspondingly.

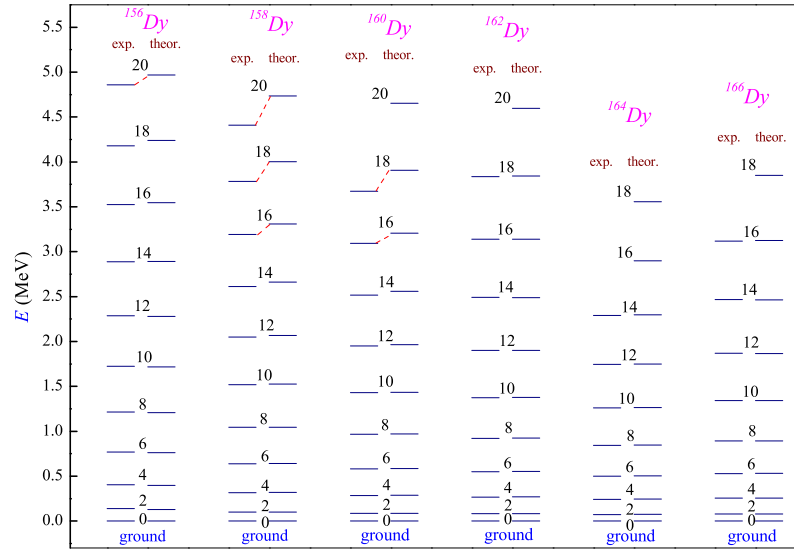


Fig. 2. The comparison experimental and theoretical spectra energy of ground bands for isotopes *Sm* and *Dy*, correspondingly.

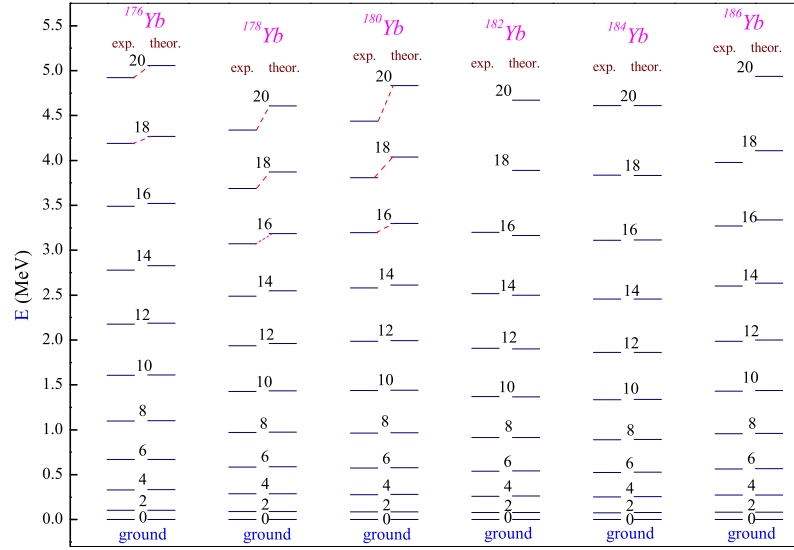


Fig. 3. The comparison experimental and theoretical spectra energy of ground bands for isotopes Sm and Yb , correspondingly.

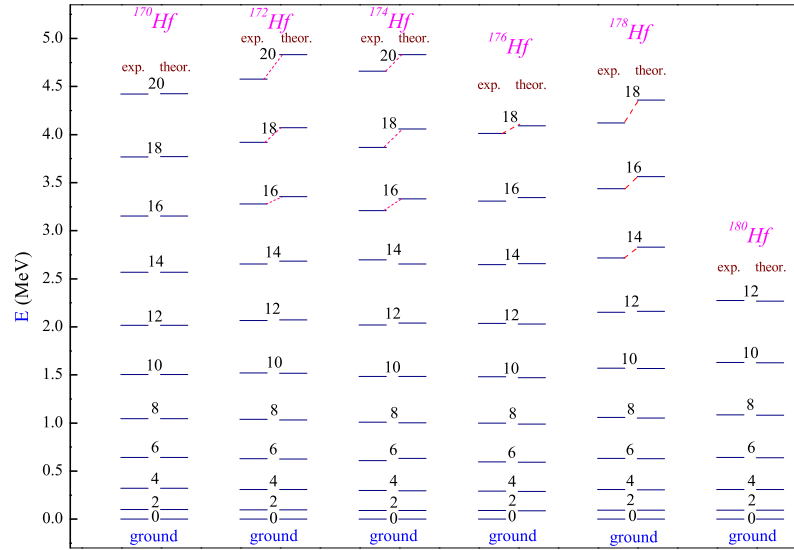


Fig. 4. The comparison experimental and theoretical spectra energy of ground bands for isotopes Sm and Hf , correspondingly.

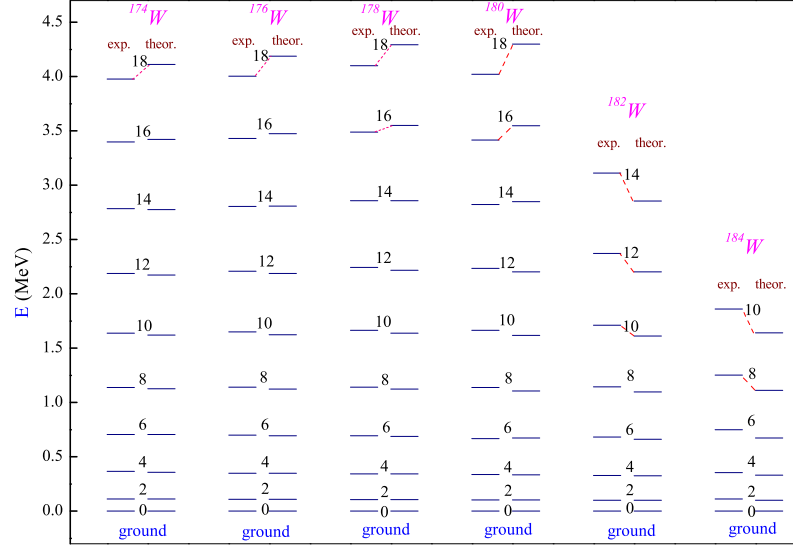


Fig. 5. The comparison experimental and theoretical spectra energy of ground bands for isotopes Sm and W , correspondingly.

4 CONCLUSIONS AND FURTHER STUDIES

This work is based on the phenomenological model [2, 3], which clearly describes large number of experimental data by the deviation properties of the positive parity in even-even deformed nuclei from the role of adiabatic theory. Spectral energy of ground states for the isotopes $^{152-156}Sm$, $^{156-166}Gd$, $^{156-166}Dy$, $^{166-176}Yb$, $^{172-182}Hf$ and $^{172-176}W$ were calculated show the violation in the $E(I) \sim I(I+1)$ law. This is explained by the fact that the nuclear core under rotation with the large mixture frequency of ground-state bands with

other rotational bands that have vibrational characters. The calculation takes into account the Coriolis mixing of positive parity states which has good agreement with experimental data.

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